5.3 Divergence Test Cont.

Monday, September 5, 2022

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Objectives

1. Mastering the basics of sequences & series
2. Practice determining if a seq. or ser. diverges or converges.
3. If a seq. or series converges, practice determining the value of convergence.
4. Discovery Worksheet

Previously...

1. Infinite sequence:

$a_n \to$ general term

$\{a_n\} \to$ sequence

$n \to$ nth term or the nth index

The sequence converges if $\lim_{n\to\infty} a_n$ exists.

2. Infinite series:

$$\underbrace{\sum_{n=1}^{\infty}}_{n \text{ index}} \underbrace{a_n}_{\text{general term of sequence}} = \overbrace{a_1}^{\text{1st term}} + \overbrace{a_2}^{\text{2nd term}} + \overbrace{a_3}^{\text{3rd term}} + \ldots$$

- Note the series does not have to start at $n=1$.

3. Kth partial sum:

1st term $\to S_1 = \sum_{n=1}^{1} a_n = a_1$

1st to 2nd term $\to S_2 = \sum_{n=1}^{2} a_n = a_1 + a_2$

3

1st to
2nd term

$$S_3 = \sum_{n=1}^{3} a_n = a_1 + a_2 + a_3$$

1st to
3rd term

$\vdots$

1st term 2nd term

$$S_K = \sum_{n=1}^{K} a_n = a_1 + a_2 + a_3 + \ldots + a_K$$

1st to
Kth term

↳ does not have to start at $n=1$.

4. Infinite Sum:

$$S = \lim_{K \to \infty} \sum_{n=1}^{K} a_n$$

If limit exists, it converges.

If limit DNE, it diverges.

5. Convergence theorem:

series

If $\lim_{K \to \infty} \sum_{n=1}^{K} a_n$ converges,

then the $\lim_{n \to \infty} a_n = 0$.

sequence

- Note that the converse does not hold.

6. Divergence Test:

sequence seq.

If $\lim_{n \to \infty} a_n \neq 0$ or $\lim_{n \to \infty} a_n \to$ DNE,

then the series $\sum_{n=1}^{\infty} a_n$ diverges.

- Note if $\lim_{n \to \infty} a_n = 0$, then the divergence test is inconclusive.

7. Sequence characteristics:

a. Arithmetic → common diff.
b. Geometric → common ratio.
c. neither Arithmetic nor geometric → find algebraic pattern.

8. Special types of series:

A. Geometric series:

$$\sum_{n=1}^{\infty} a_1 r^{n-1}$$

r ↓ common ratio

Why?
- Convergence & divergence
  - If $|r|<1$, then the series converges
  - If $|r|=1$, then the series diverges
  - If $|r|>1$, then the series diverges

- kth partial sum

why?

$$S_k = \sum_{n=1}^{k} a_1 r^{n-1} = \frac{a_1(1-r^k)}{1-r} \quad \text{for } r \neq 1$$

- Infinite sum if the series converges

$$S = \lim_{k\to\infty} \sum_{n=1}^{k} a_1 r^{n-1} = \frac{a_1}{1-r} \quad \text{for } r \neq 1$$

why

B. telescoping Series:

→ the finite sums in which consecutive pairs cancel each other, leaving only the initial and final terms.

initial and final terms.

In general for telescoping sums - if it converges,

$$\underset{\text{finite upper term}}{\longrightarrow} \sum_{n=2}^{N} (a_n - a_{n-1}) = a_N - a_1$$

final term

first term

seq. gen. term

previous term

- convergence & divergence

  - Telescoping series usually converges but not always
  - How to check if the sequence converges?

  1. Compute the kth partial sum general term.

$$S_1 = \sum_{n=1}^{1} a_n$$

$$S_2 = \sum_{n=1}^{2} a_n$$

$$\vdots$$

$$S_k = \sum_{n=1}^{k} a_n$$

Find the pattern of the sequence of partial sums.

- Note that this technique can be used to any series - not just telescoping.

  2. Determine if $\lim_{k \to \infty} \sum_{n=1}^{k} a_n$ exists.

Group Activity

- Geometric Series Discovery Worksheet
- Assigned groups
  - 13 students, 4 groups
  - 3-4 members in each group

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- For each problem in the worksheet,
  Discuss within your group;
  a. the objective of the problem
  b. the facts of the problem
  c. the strategies to solve the problem
  d. agree on the final solution and write it on the worksheet.

- Submit your group work by end-of-class.

- Randomly generated groups for this week.

✓ group 1: Kian C., Malory M., Jarod T.
✓ group 2: Evan Ta., Taylor W., Jonathan N.
✓ group 3: Nico C., Emma P., Evan Yo.
✓ group 4: Niksela R., Romie DS., Ryan O., Nathan PF.

today's leaders

- Leaders are responsible for
  + writing the group's agreed solution.
  + submitting the worksheet.