3.2 Trigonometric Integrals Cont.

Friday, October 7, 2022

Objectives:

1. Products and powers of tan(x) and sec(x) integrals.

Products and powers of tan(x) and sec(x)

Integrals of the form:

$$\int \tan^k(x)\sec^j(x)\,dx$$

Strategies:

1. If j is an even number and $j \geq 2$, rewrite $\sec^j(x) = \sec^{j-2}(x)\sec^2(x)$
   and use $\sec^2(x) = \tan^2(x) + 1$
   to rewrite $\sec^{j-2}(x)$ in terms of $\tan(x)$.
   Use u-substitution $u = \tan(x)$ and $du = \sec^2(x)\,dx$.

2. If k is an odd number and $j \geq 1$, rewrite $\tan^k(x)\sec^j(x) = \tan^{k-1}(x)\sec^{j-1}(x)\sec(x)\tan(x)$
   and use $\tan^2(x) = \sec^2(x) - 1$
   to rewrite $\tan^{k-1}(x)$ in terms of $\sec(x)$.
   Use u-substitution $u = \sec(x)$ and $du = \sec(x)\tan(x)\,dx$

3. If j is even and k is odd, use 1 or 2.

4. If k is odd where $k \geq 3$ and $j = 0$, rewrite $\tan^k(x) = \tan^{k-2}(x)\tan^2(x)$
   $= \tan^{k-2}(x)(\sec^2(x) - 1)$
   $\tan^k(x) = \tan^{k-2}(x)\sec^2(x) - \tan^{k-2}(x)$

5. If k is even and j is odd, then use $\tan^2(x) = \sec^2(x) - 1$
   to express $\tan^k(x)$ in terms of $\sec(x)$.
   Use integration by parts to integrate odd powers of $\sec(x)$.

Example:

- $\int \tan^{\overset{\text{even}}{6}}(x)\sec^{\overset{\text{even}}{4}}(x)\,dx$

  rewrite $\sec^4(x) = \sec^2(x)\sec^2(x)$ and $\sec^2(x) = \tan^2(x) + 1$

$$= \int \tan^6(x)(\tan^2(x)+1)\sec^2(x)\,dx$$

let $u = \tan(x)$ and $du = \sec^2(x)\,dx$

$$= \int u^6(u^2+1)\,du$$

$$= \int (u^8 + u^6)\,du$$

$$= \frac{u^9}{9} + \frac{u^7}{7} + C$$ → put back $u = \tan(x)$

$$= \int (u^8 + u^6)\,du$$

$$= \frac{u^9}{9} + \frac{u^7}{7} + C \quad \rightarrow \text{put back } u = \tan(x)$$

$$= \frac{\tan^9(x)}{9} + \frac{\tan^7(x)}{7} + C$$

- $\int \overset{\nearrow \text{odd}}{\tan^5(x)} \overset{\nearrow \text{odd}}{\sec^3(x)}\,dx$

rewrite $\tan^5(x)\sec^3(x) = \tan^4(x)\sec^2(x)\sec(x)\tan(x)$

$$= \int (\tan^2(x))^2 \sec^2(x)\sec(x)\tan(x)\,dx$$

use $\tan^2(x) = \sec^2(x) - 1$

$$= \int (\sec^2(x) - 1)^2 \sec^2(x)\sec(x)\tan(x)\,dx$$

Let $u = \sec(x)$ and $du = \sec(x)\tan(x)\,dx$

$$= \int (u^2 - 1)^2 u^2\,du$$

$$= \int (u^6 - 2u^4 + u^2)\,du$$

$$= \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C \quad \rightarrow \text{put back } u = \sec(x)$$

$$= \frac{\sec^7(x)}{7} - \frac{2}{5}\sec^5(x) + \frac{1}{3}\sec^3(x) + C$$

- $\int \overset{\nearrow \text{odd}}{\tan^3(x)}\,dx$

rewrite $\tan^3(x) = \tan(x)\tan^2(x)$ and $\tan^2(x) = \sec^2(x) - 1$

$$= \int (\tan(x)\sec^2(x) - \tan(x))\,dx$$

$$= \int \tan(x)\sec^2(x)\,dx - \int \tan(x)\,dx$$

Let $u = \tan(x)$ and $du = \sec^2(x)\,dx$

$$= \int u\,dx - \int \tan(x)\,dx$$

$$= \frac{u^2}{2} - \ln(|\sec(x)|) + C$$

$$= \frac{\tan^2(x)}{2} - \ln(|\sec(x)|) + C$$

Mini-Activities

Today, we work on the trigonometric integrals worksheet.