

3.4 Partial Fractions

Friday, October 14, 2022

Objectives:

1. Introduction to Partial Fractions
2. Partial fractions worksheet

Partial Fractions Decomposition

Objective: decompose $\frac{5x-4}{x^2-x-2}$ into partial fractions

step 1: $\frac{5x-4}{x^2-x-2} = \frac{5x-4}{(x-2)(x+1)}$ factor denominator

step 2: $\frac{5x-4}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$ write partial fractions with constant A & B.

step 3.

$$5x-4 = A(x+1) + B(x-2) \quad \text{multiply both sides by } (x-2)(x+1)$$

step 4: Find A & B.

Substituting the roots, or "zeros" of $(x-2)(x+1)$.

Root for $(x+1)$ is $x=-1$:

$$\begin{aligned} 5(-1)-4 &= A(-1+1) + B(-1-2) \\ -9 &= 0 + B(-3) \\ B &= 3 \end{aligned}$$

Root for $(x-2)$ is $x=2$:

$$\begin{aligned} 5(2)-4 &= A(2+1) + B(2-2) \\ 6 &= A(3) + 0 \\ A &= 2 \end{aligned}$$

$$6 = A3 + 0$$
$$A = 2$$

Step 5: $\frac{5x-4}{x^2-x-2} = \frac{2}{x-2} + \frac{3}{x+1}$

Integration by Partial Fractions

• $\int \frac{5x-4}{x^2-x-2} dx = \int \left(\frac{2}{x-2} + \frac{3}{x+1} \right) dx$

partial fractions decomposition \rightarrow

$$= \int \frac{2}{x-2} dx + \int \frac{3}{x+1} dx$$
$$= 2 \ln|x-2| + 3 \ln|x+1| + C$$

Group Activity:

Partial Fractions Worksheet.