3.4 Partial Fractions Cont. Cont.

Wednesday, October 26, 2022

Objectives

1. long division before partial fraction decomposition
2. Partial fractions for irreducible quadratic factor.

Previously...

Revisit this problem

- $\int \frac{x-3}{x+2}\,dx$

PFD: $\frac{x-3}{x+2} = \frac{A}{1} + \frac{B}{x+2}$

$x-3 = A(x+2) + B$

root of $x+2$ is $x+2=0 \to x=-2$.

solve for B: $-2-3 = B \to B = -5$

solve for A: $x-3 = A(x+2) - 5$

apply $x=1$: $1-3 = 3A - 5$

$-2+5 = 3A$

$3 = 3A$

$A = 1$

this might not work in every case, so scratch this.

$\int \frac{x-3}{x+2}\,dx = \int \left(1 - \frac{5}{x+2}\right) dx$

$= \int dx - \int \frac{5}{x+2}\,dx$

$$= x - 5\ln(x+2) + C$$

Long Division before partial fractions

- $\displaystyle\int \frac{x^2+3x+5}{x+1}\,dx$

numerator : $\deg(x^2+3x+5) = 2$
denominator : $\deg(x+1) = 1$

$$\deg(x^2+3x+5) \geq \deg(x+1)$$

we need to do long division first and then partial fractions next if needed.

The process

$$\frac{x^2+3x+5}{x+1}$$

$$\begin{array}{r|l} & x+2+\dfrac{3}{x+1} \\ \hline x+1 & x^2+3x+5 \\ & -(x^2+x) \\ \hline & 0+2x+5 \\ & -(2x+2) \\ \hline & 0+3 \end{array}$$

$\dfrac{x^2}{x} = x$

$x(x+1) = x^2+x$

$\dfrac{2x}{x} = 2$

$2(x+1) = 2x+2$

$\downarrow$

remainder $\dfrac{3}{x+1}$

So, $\dfrac{x^2+3x+5}{x+1} = x+2+\dfrac{3}{x+1}$

$$\int \frac{x^2+3x+5}{x+1}\,dx = \int \left(x+2+\frac{3}{x+1}\right)dx$$

$$= \int x\,dx + \int 2\,dx + \int \frac{3}{x+1}\,dx$$

$$= \frac{x^2}{2} + 2x + 3\ln(x+1) + C$$

- $\displaystyle\int \frac{x-3}{x+2}\,dx$

$\deg(x-3) = \deg(x+2)$

we can do long division

$\frac{x-3}{x+2}$

$$\begin{array}{r|l} & 1 - \frac{5}{x+2} \\ \hline x+2 & x-3 \\ & -(x+2) \\ \hline & 0-5 \end{array}$$

$\frac{x}{x} = 1$

$1(x+2)$

$\downarrow$

remainder $\frac{-5}{x+2}$

$$\int \frac{x-3}{x+2}\,dx = \int \left(1-\frac{5}{x+2}\right)dx$$

$$= x - 5\ln(x+2) + C$$

Partial Fractions with irreducible factors

- $\displaystyle\int \frac{2x-3}{x^3+x}\,dx$

- $\int \frac{2x-3}{x^3+x}\,dx$

$$\deg(2x-3) < \deg(x^3+x)$$

we can do partial fraction decomposition

$$\frac{2x-3}{x^3+x} = \frac{2x-3}{x(x^2+1)}$$

↪ irreducible

PFD:

$$\frac{2x-3}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$2x-3 = A(x^2+1) + (Bx+C)x$$

$$= Ax^2 + A + Bx^2 + Cx$$

$$= Ax^2 + Bx^2 + Cx + A$$

$$0x^2 + 2x - 3 = (A+B)x^2 + Cx + A$$

$$\downarrow$$

$$0 = A + B \rightarrow 0 = -3 + B$$

$$2 = C \qquad B = 3$$

$$-3 = A$$

$$\int \frac{2x-3}{x^3+x}\,dx = \int \left(\frac{-3}{x} + \frac{3x+2}{x^2+1} \right) dx$$

$$= -\int \frac{3}{x}\,dx + \int \frac{3x+2}{x^2+1}\,dx$$

$$= -\int \frac{3}{x}\,dx + \int \frac{3x}{x^2+1} + \int \frac{2}{x^2+1}\,dx$$

by u-sub   by using standard integral guide

$$= -3\ln(x) + \frac{3}{2}\ln(x^2+1) + 2\tan^{-1}(x) + C$$

Mini-Assignment

1. $\int \frac{2}{(x-4)(x^2+2x+6)}\,dx$

2. $\int \frac{x^3+6x^2+3x+6}{x^3+2x^2}\,dx$