2.1 Area Between Curve Cont. & 2.2 Volume by Slicing

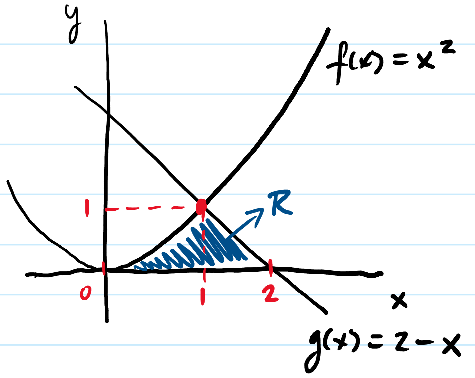
Wednesday, November 2, 2022

Objectives:

1. Area of a region between two curves integrating with respect to the dependent variable.
2. Determine the volume of a solid.
   a. Slicing method

Integrating with respect to x

Ex.



Intersection:

$$x^2 = 2 - x$$
$$x^2 + x - 2 = 0$$
$$(x-1)(x+2) = 0$$
$$x = 1, \quad x = -2$$

@ A_1, $x = [0, 1]$, $f(x) = x^2$

@ A_2, $x = [1, 2]$, $g(x) = 2 - x$

$$A_1 = \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$A_2 = \int_1^2 (2-x)\,dx = 2x - \frac{x^2}{2}\Big|_1^2 = \frac{1}{2}$$

total Area: $A = A_1 + A_2 = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

Integrating with respect to y

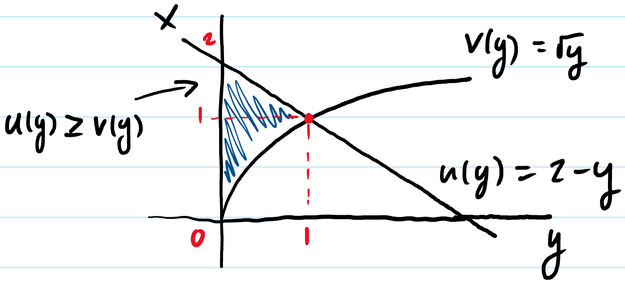
Integrating with respect to y

Ex. $f(x) = x^2$ and $g(x) = 2 - x$

$y = x^2$ $\qquad$ $y = 2 - x$

$\sqrt{y} = x$ $\qquad$ $2 - y = x$

$x = v(y) = \sqrt{y}$ $\qquad$ $x = u(y) = 2 - y$

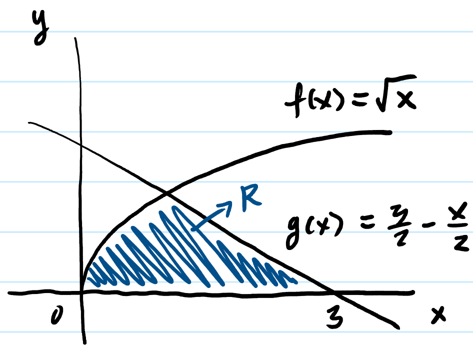


$$A = \int_0^1 (2 - y - \sqrt{y})\, dy$$

$$= 2y - \frac{y^2}{2} - \frac{2}{3} y^{3/2} \Big|_0^1$$

$$A = \frac{5}{6}$$

Mini-Assignment part 1



Determine the area of R
by integrating with respect to y.

Volume of Solids using the Slice method

- solid in 3D Example

A pyramid: Recall that the formula is $V_{pyramid} = \frac{1}{3} a^2 h$ for a square base

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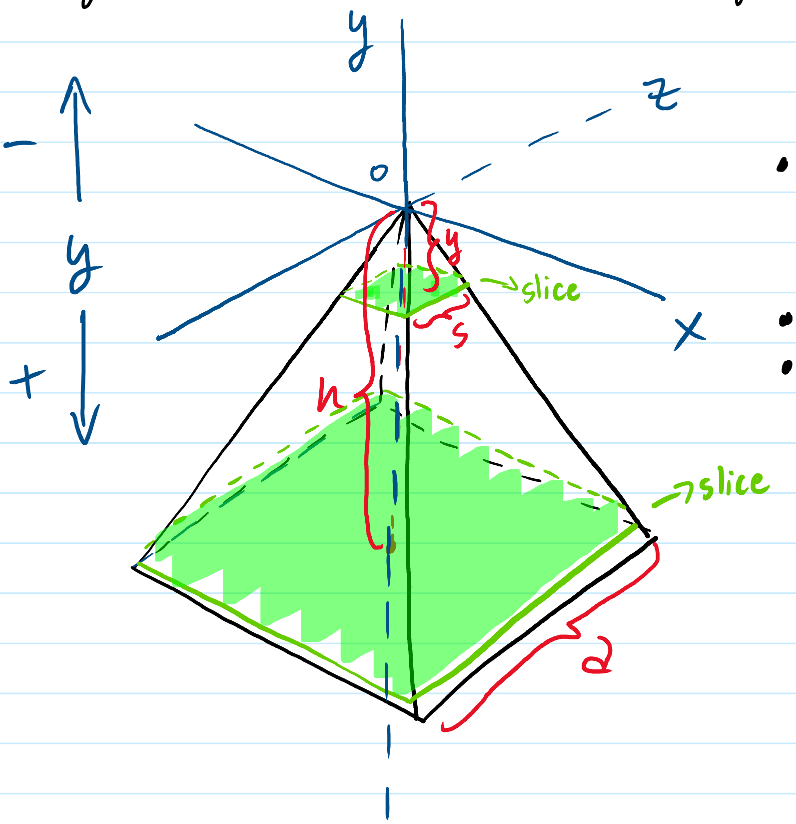


• A square base with a as the length of one side

• Area of a square with side s is s^2.

• Since the base is a square, then the cross-sections are also square.

Using similar triangles, area for each cross-section is

$$\frac{s}{a} = \frac{y}{h}$$

$$s = \frac{ay}{h}$$

$$A(y) = s^2 = \left(\frac{ay}{h}\right)^2$$

$A(y)$ → area of cross-sectional squares

$$V = \int_0^h A(y)\,dy$$

$$= \int_0^h \left(\frac{ay}{h}\right)^2 dy$$

$$= \frac{a^2}{h^2} \int^h y^2\,dy$$

$$= \frac{a^2}{h^2} \int_0^h y^2 \, dy$$

$$= \frac{a^2}{h^2} \frac{y^3}{3} \Big|_0^h$$

$$= \frac{a^2}{h^2} \frac{h^3}{3}$$

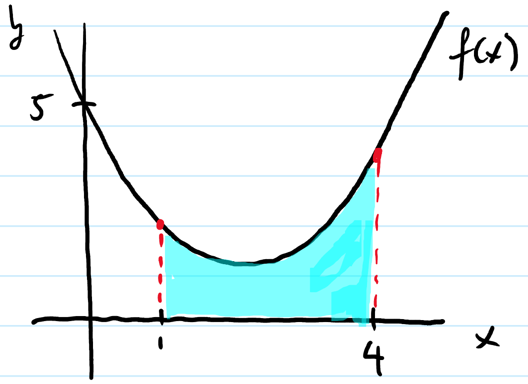
$$V = \frac{1}{3} a^2 h \quad \checkmark$$

Volume of a Solid of Revolution Using the Slicing method

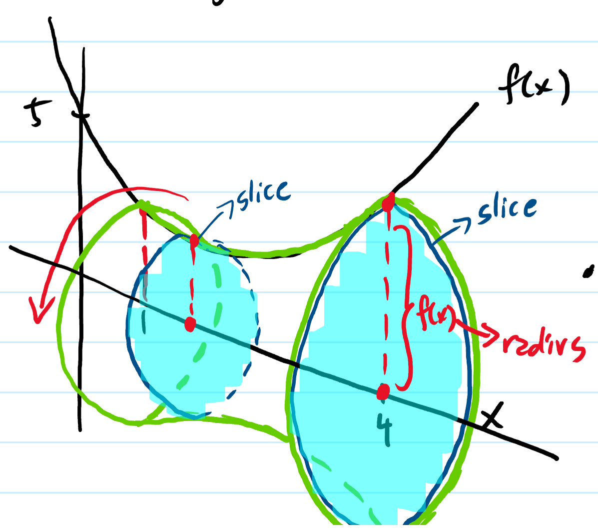
Solid is defined by these functions

$f(x) = x^2 - 4x + 5$, $x = 1$, and $x = 4$

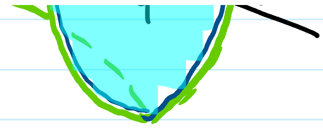
first:



second: Revolve the graph around the x-axis



- Area of a circle with some radius r.



Area of a circle as a function of x.

$$A(x) = \pi r^2 = \pi (f(x))^2 = \pi (x^2 - 4x + 5)^2$$

Find the volume

$$V = \int_a^b A(x)\, dx$$

$$= \int_1^4 \pi (x^2 - 4x + 5)^2\, dx$$

$$= \int_1^4 \pi (y^4 - 8x^3 + 26x^2 - 40x + 25)\, dx$$

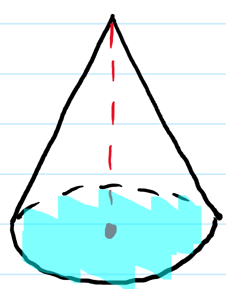
$$= \pi \left(\frac{x^5}{5} - 2x^4 + \frac{26x^3}{3} - 20x^2 + 25x \right) \Bigg|_1^4$$

$$V = \frac{78}{5} \pi .$$

Mini-Assignment Part 2

1. Use the slicing method to derive the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a circular cone.

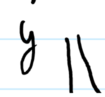
rough sketch



   a. draw the solid with axis labels and variables, and cross sections.
   b. Set-up and solve the integral.

2. Find the volume of the solid defined by the function $f(x) = \frac{1}{x}$ revolved around the x-axis over the interval $[1, 2]$.

rough sketch



over the interval $[1,2]$.

a. Sketch the solid with axis labels.
b. Set-up and solve the integral.