2.2 Volume by Slicing Cont. & 2.3 Cylindrical Shells

Monday, November 7, 2022

Objectives:

1. Continue on finding volume of solid of revolution, using the washer method.
2. Introduction to the shell method.

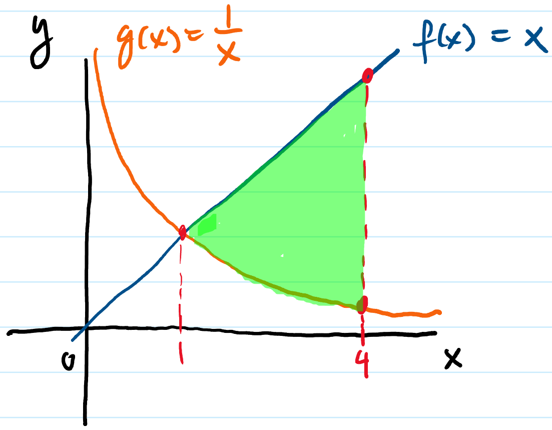
The washer method

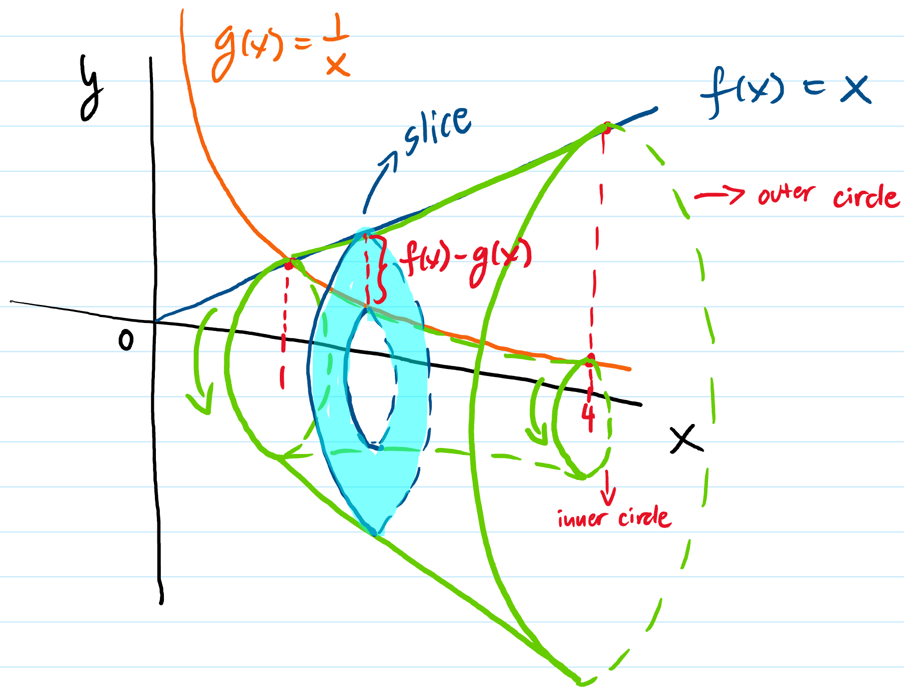
Given continuous functions $f(x)$ and $g(x)$, $f(x) \geq g(x)$ over the interval $[a,b]$.

$$V = \int_a^b \pi\left[(f(x))^2 - (g(x))^2\right] dx$$

Example:

Suppose we have a solid bounded by the functions $f(x) = x$ and $g(x) = 1/x$ over the interval $[1,4]$, and revolved around the x-axis.





$$V = \int_a^b \pi\left(f(x)^2 - g(x)^2\right) dx$$

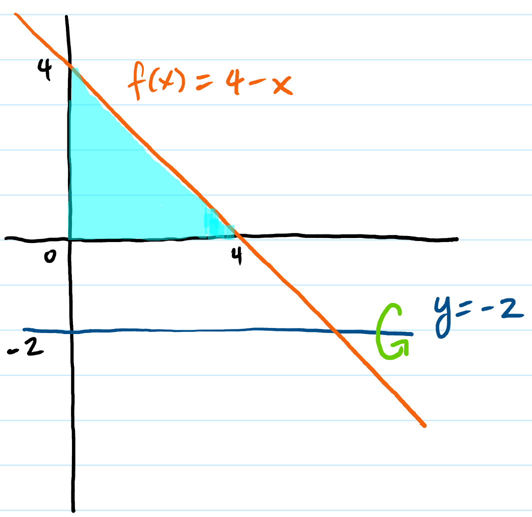
$$= \int_1^4 \pi\left(x^2 - \left(\frac{1}{x}\right)^2\right) dx$$

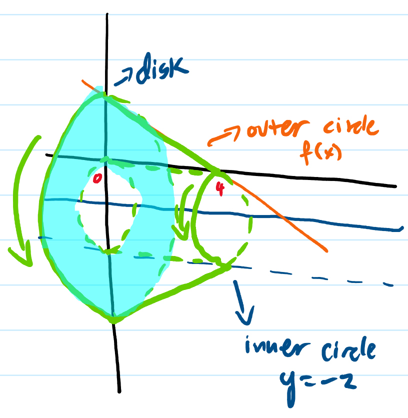
$$= \pi\left(\frac{x^3}{3} + \frac{1}{x}\right)\Big|_1^4$$

$$V = \frac{81\pi}{4}$$

The washer method with different axis of revolution

revolve around $y=-2$





Volume of a solid revolve around $y=-2$.

- radius of outer circle: $f(x) + 2 = (4-x)+2 = 6-x$.
- radius of inner circle: $g(x) = 2$

$$V = \int_0^4 \pi\left((6-x)^2 - (2)^2\right) dx$$

$$= \pi\left(\frac{x^3}{3} - 6x^2 + 32x\right)\Big|_0^4$$

$$V = \frac{160\pi}{3}$$

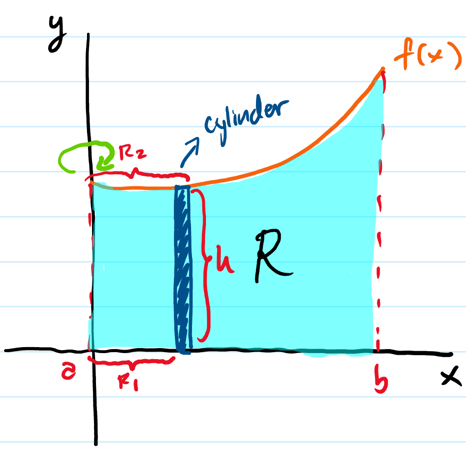
Mini-Activity Part 1

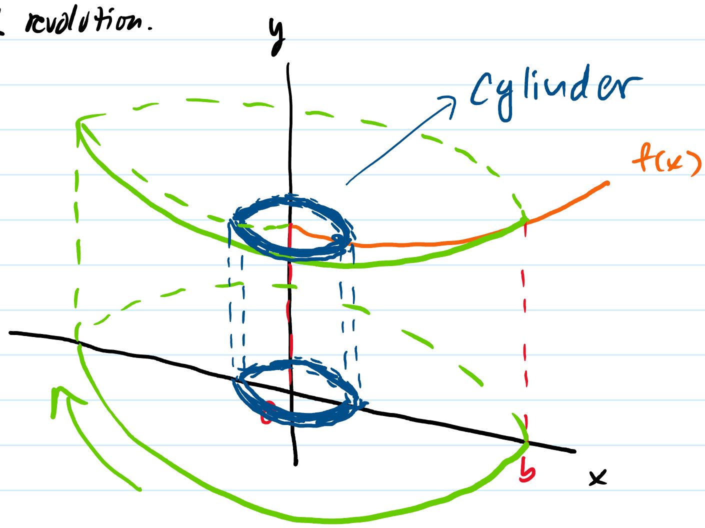
1. Find the volume of the solid using the washer method revolved around the x-axis bounded by the functions $f(x) = \sqrt{x}$ and $g(x) = 1/x$ over the interval $[1,3]$. Sketch the solid
2. Find the volume of the solid using the washer method revolved around $y=-1$ bounded by the functions $f(x) = x+2$ over the interval $[0,3]$. Sketch the solid.

Cylindrical Shells

Recall: Disk and washer methods are ways to find volumes of solid of revolution integrating along the axis parallel to the axis of revolution.

Shell method: we integrate along the coordinate axis perpendicular to the axis of revolution.





revolve around the y-axis.

Volume of a cylinder: $V_{cylinder} = \pi r^2 h$

$r \downarrow$ radius, $h \downarrow$ height

Cylindrical shells with two radius: $r_1 \to$ outer radius
$r_2 \to$ inner radius

$$V_{shell} = \pi r_1^2 h - \pi r_2^2 h$$

$$V_{shell} = 2\pi \underbrace{\left(\frac{r_2+r_1}{2}\right)}_{r} h \underbrace{(r_2 - r_1)}_{\Delta r}$$

$$V_{shell} = 2\pi r h \Delta r$$

In terms of x:

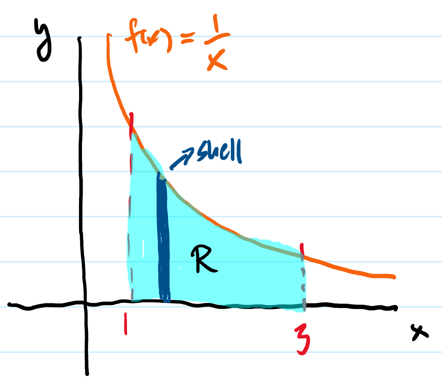
$$V_{shell} = 2\pi x f(x) \Delta x$$

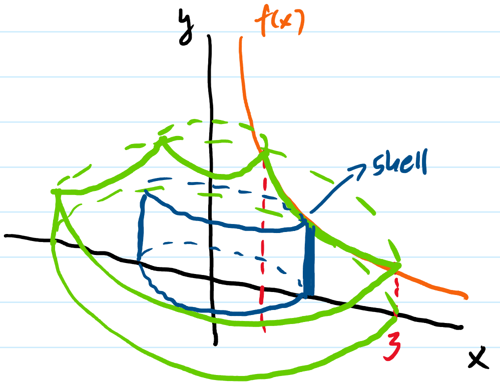
Assume $f(x)$ is continuous, then

$$V = \int_a^b 2\pi x f(x)\, dx$$

Example:

Find the volume of the region using the shell method revolved around the y-axis bounded by $f(x) = 1/x$ over the interval $[1,3]$.





$$V = \int_1^3 2\pi x \left(\frac{1}{x}\right) dx$$

$$= \int_1^3 2\pi \, dx$$

$$= 2\pi x \Big|_1^3$$

$$V = 4\pi$$

Mini-Activity Part 2

3. Find the volume of the region bounded above by $f(x)=x^2$ and below by the x-axis over the interval $[1,2]$, revolved around the y-axis. Sketch the solid.