2.3 Cylindrical Shells Cont.

Wednesday, November 9, 2022

Objectives:

1. Continue on finding volumes of solid of revolution using the shell method.
2. Shell method for solids revolving around the y-axis, x-axis, and around a line.

Previously...

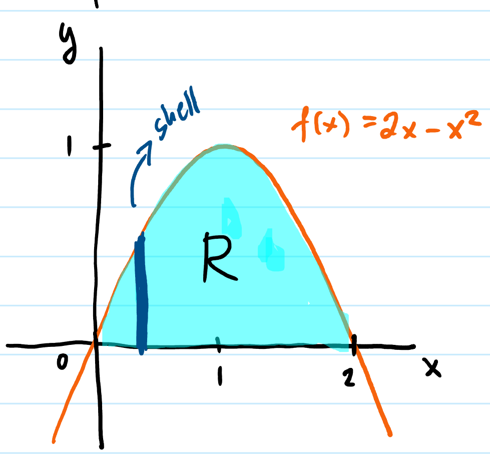
The method of cylindrical shells revolved around the y-axis.

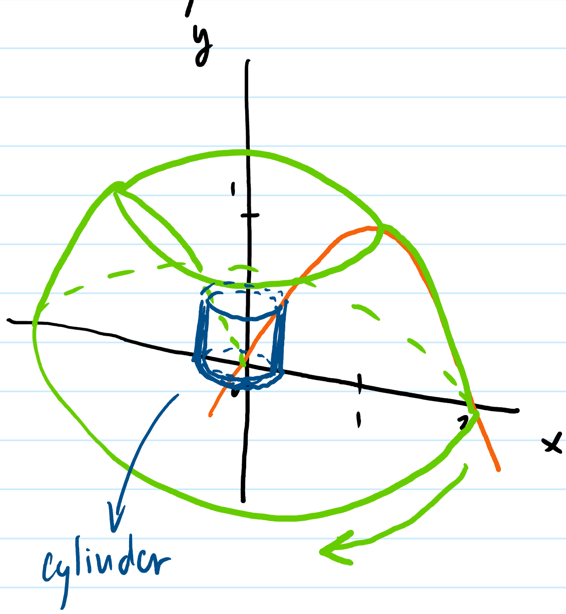
Define a region R bounded above by f(x) below by the x-axis and x=a on the left and x=b on the right. The volume of solid of revolution is

$$V=\int_a^b 2\pi x f(x)\,dx$$

where f(x) is continuous and nonnegative.

Example:





$$V=\int_0^2 2\pi x(2x-x^2)\,dx$$

$$V = 2\pi \int_0^2 (2x^2 - x^3)\, dx$$

$$= 2\pi \left(\frac{2x^3}{3} - \frac{x^4}{4} \right) \Big|_0^2$$

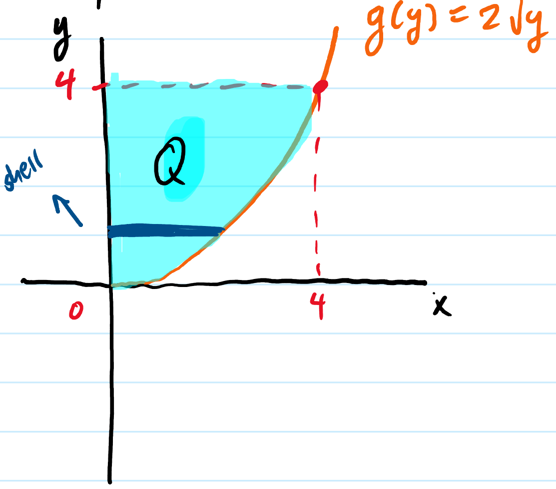
$$V = \frac{8\pi}{3}$$

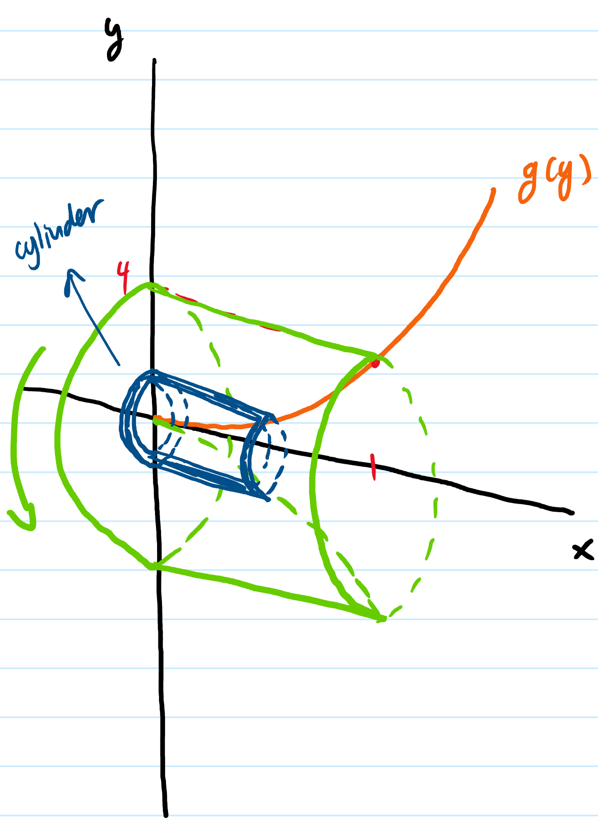
Cylindrical shells method revolved around the x-axis

Define Q as the region bounded on the right by $g(y)$, on the left by the y-axis, below by the line $y = c$, and above by the line $y = d$.

$$V = \int_c^d 2\pi y\, g(y)\, dy.$$

Example:





$$V = \int_0^4 2\pi y\, 2\sqrt{y}\, dy$$

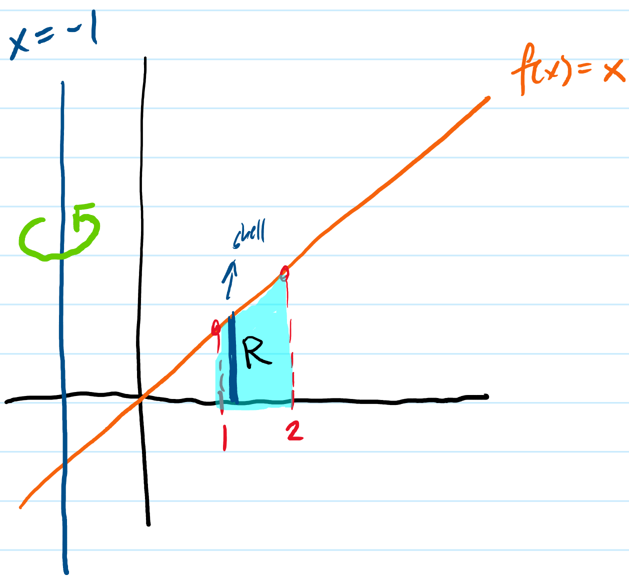
$$= 4\pi \int_0^4 y^{3/2}\, d\ldots$$

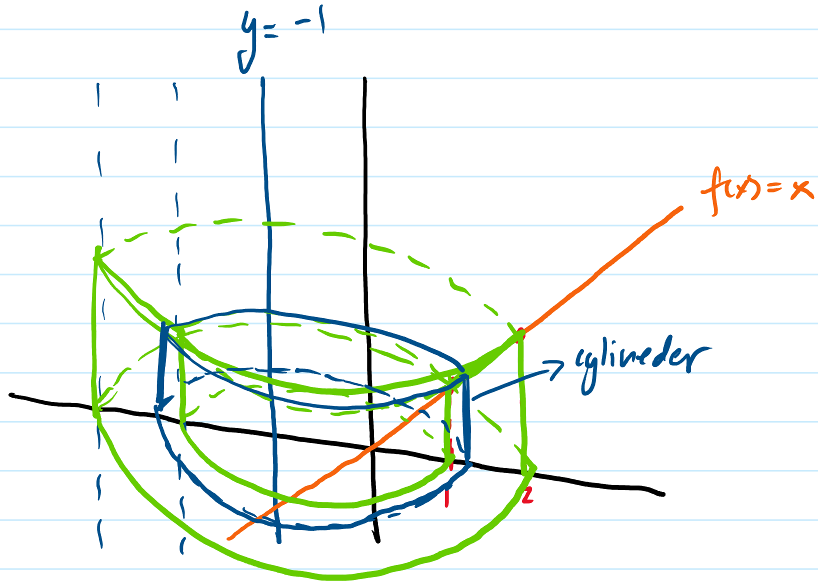
$$= 4\pi\int_0^4 y^{3/2}\,dy$$

$$V = 4\pi\left(\frac{2y^{5/2}}{5}\right)\Big|_0^4$$

$$V = \frac{256\pi}{5}$$

Revolution Around a line





f(x)

add 1 because we revolve it around x = -1.

radius of the shell is $x+1$.

$$V = \int_a^b 2\pi(x+1)f(x)\,dx$$

$$= \int_1^2 2\pi(x+1)x\,dx$$

$$= 2\pi\int_1^2 (x^2+x)\,dx$$

$$= 2\pi\left(x^3 + x^2\right)\Big|^2$$

$$= 2\pi\left(\frac{x^3}{3} + \frac{x^2}{2}\right)\Big|_1^2$$

$$V = \frac{23\pi}{3}$$

Mini-Activities

1. Define R as the region bounded above by the graph $f(x) = 3x - x^2$ and below by the x-axis over the interval $[0,2]$. Find the volume of the solid of revolution formed by revolving around the y-axis. Sketch the solid.

2. Define Q as the region bounded on the right by the graph $g(y) = 3/y$ and on the left by the y-axis for $y \in [1,3]$. Find the volume of the solid of revolution formed by revolving around the x-axis. Sketch the solid.

3. Define R as the region bounded above by the graph $f(x) = x^2$ and below by the x-axis over the interval $[0,1]$. Find the volume of the solid of revolution by revolving around the line $x = -2$. Sketch the solid.