7.1 Parametric Equations

Monday, November 28, 2022

Objectives:

1. Introduce parametric equations.
2. Plot a curve described by parametric equations
3. Parametric equations of basic curves, lines and circles, and cycloid.

Parametric Equations

If x and y are continuous functions of t on an interval I, then the equations

$$x = x(t) \quad \text{and} \quad y = y(t)$$

are called parametric equations and t is called a parameter.
The graph of parametric equations is called a parametric curve or plane curve, and is denoted by C.
The set of point (x, y) obtained as t is called the graph.

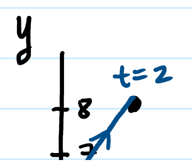
Examples:

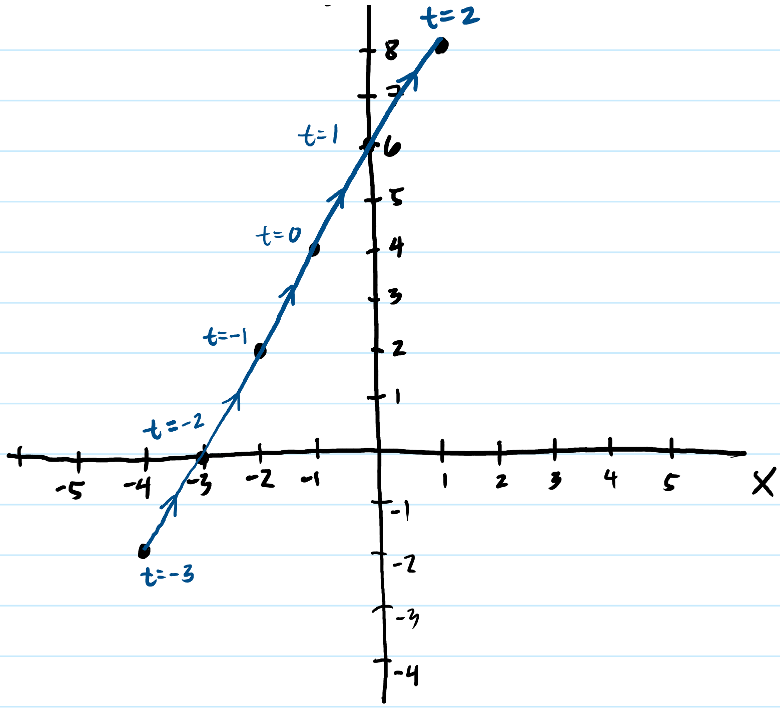
- $x(t) = t - 1$, $y(t) = 2t + 4$, $-3 \le t \le 2$

This is an example of a parametric line.

Table of values or points on the curve.

t	x(t)	y(t)
-3	-4	-2
-2	-3	0
-1	-2	2
0	-1	4
1	0	6
2	1	8

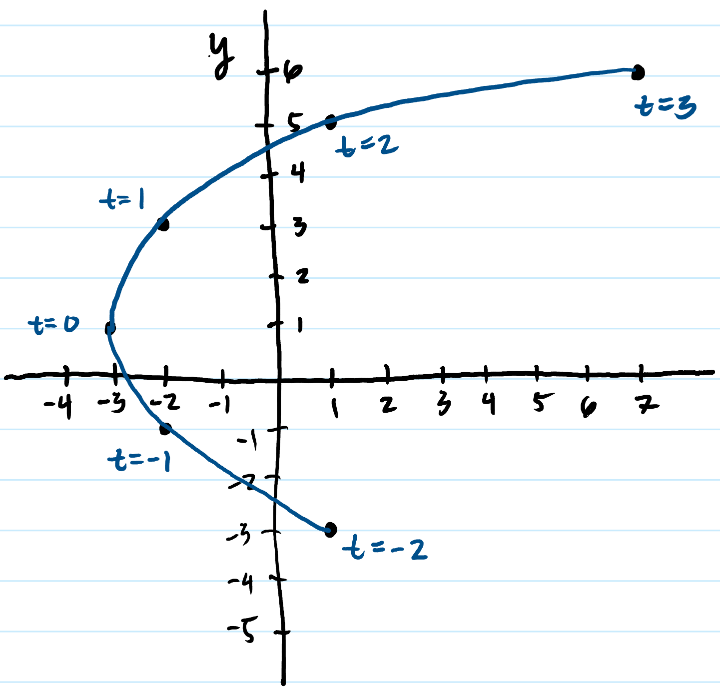




- $x(t) = t^2 - 3, \; y(t) = 2t + 1, \; -2 \le t \le 3$

this is an example of a parametric parabola.

t	$x(t)$	$y(t)$
-2	1	-3
-1	-2	-1
0	-3	1
1	-2	3
2	1	5
3	6	7



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- You can also use technology

Desmos Demonstration:

1st example: enter: $(t-1, 2t+4)$
$-3 \le t \le 2$

2nd example: enter: $(t^2-3, 2t+1)$
$-2 \le t \le 3$

Mini-Activity Part 1

Graph these parametric equations using Desmos.

1. $x(t) = 4\cos(t)$, $y(t) = 4\sin(t)$, $0 \le t \le 2\pi$ $\longrightarrow$ this is an example of a parametric circle.
2. $x(t) = 3t+2$, $y(t) = t^2-1$, $-3 \le t \le 2$

Eliminating a Parameter

Goal:

parametric
$x = x(t)$
$y = y(t)$
$\xrightarrow{\text{turn into}}$
functions of x or y.
$y = f(x)$ or $x = g(y)$

Example:

$$x(t) = t^2 - 3, \quad y(t) = 2t+1, \quad -2 \le t \le 3$$

$x(t) = t^2-3$ $\downarrow$ solve for t $\downarrow$ $t = \sqrt{x+3}$ $\downarrow$ substitute into $y(t)$ $\downarrow$

$y(t) = 2t+1$ $\downarrow$ solve for t $\downarrow$ $t = \dfrac{y-1}{2}$ $\downarrow$ substitute into $x(t)$ $\downarrow$ $x = \left(\dfrac{y-1}{2}\right)^2 - 3$

into $y(t)$

$$y = 2\sqrt{x+3} - 3$$

$$f(x) = 2\sqrt{x+3} - 3$$

$$x = \left(\frac{y-1}{2}\right)^2 - 3$$

$$g(y) = \frac{1}{4}\left(y^2 - 2y - 11\right)$$

Parameterizing a Curve

Goal:

functions of x & y $\xrightarrow{\text{turn into}}$ parametric

$f(x)$ or $g(y)$

$x = x(t)$

$y = y(t)$

Example:

- $y = 2x^2 - 3$

Way 1: we can do this always.

$x(t) = t$, $y(t) = 2t^2 - 3$, domain $-\infty \le t \le \infty$

Way 2:

Let $\underbrace{x(t) = 3t - 2}$ $\rightarrow$ $y = 2x^2 - 3$

no domain restrictions $-\infty \le t \le \infty$

arbitrary as long as the domain and range is consistent with y.

substitute $x(t)$ for x

$$y = 2(3t-2)^2 - 3$$
$$= 18t^2 - 24t + 5$$

Thus, $x(t) = 3t - 2$ and $y(t) = 18t^2 - 24t + 5$, $-\infty \le t \le \infty$

Mini-Activity Part 2

3. Find two different sets of parametric equations to represent the graph of $y = x^2 + 2x$.
Use desmos to plot your parametric curves and y to see if they match.

Desmos Demonstrations (parametric equations)

- General Circle with radius r.

$$x(\theta) = r\cos(\theta) \qquad 0 \le \theta \le 2\pi$$
$$y(\theta) = r\cos(\theta)$$

- General Cycloids with radius a (traveling on a straight line)

$$x(t) = a(t - \sin(t))$$
$$y(t) = a(1 - \cos(t))$$

- General hypercycloid with radius a and b.

$$x(t) = (a-b)\cos(t) + b\cos\left(\frac{a-b}{b}\right)t$$

$$y(t) = (a-b)\sin(t) - b\sin\left(\frac{a-b}{b}\right)t$$