7.3 Polar Coordinates Cont.

Wednesday, December 7, 2022

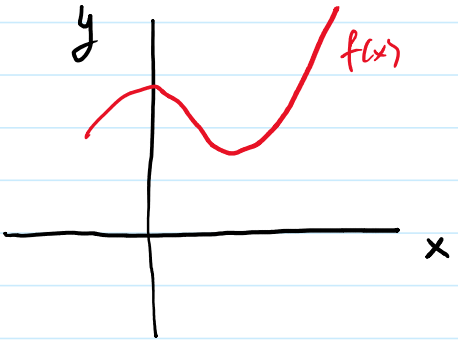
Objectives:

1. Convert equations between (x,y) to polar coordinates.
2. sketch polar curves from given equations.

Polar Curves

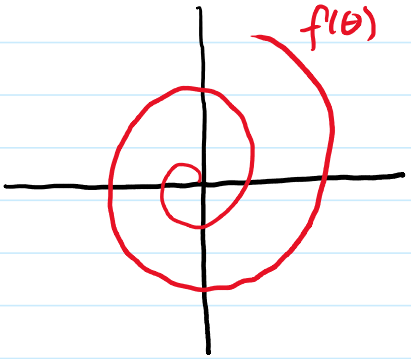
Cartesian plane

$y = f(x)$



polar coordinates

$r = f(\theta)$



Transforming Polar Equation to (x,y) coordinates

Examples:

- $r = 4\sin(\theta)$

$r(r) = r4\sin(\theta)$ → multiply by r

$r^2 = r4\sin(\theta)$

→ use $x^2+y^2=r^2$ and $y=r\sin\theta$

$x^2+y^2 = 4y$

$x^2+y^2-4y = 0$

$x^2+(y^2-4y) = 0$

↓

$x^2+(y-2)^2-4 = 0$ → complete the square

ay^2+by+c

$a=1,\ b=-4,\ c=0$

↓

$(x-0)^2+(y-2)^2 = 4$

$(x-0)^2+(y-2)^2 = 2^2$

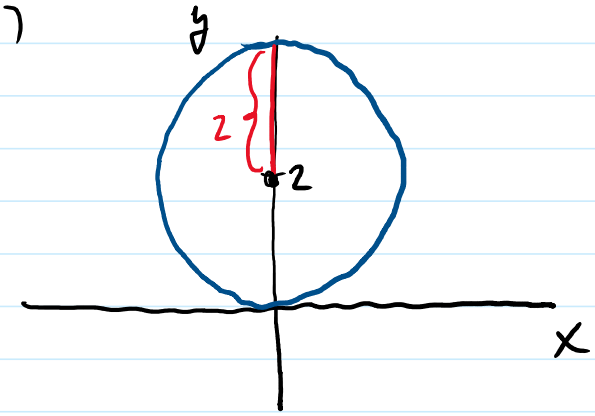
$(x-0)^2 + (y-2)^2 = 4$

$(x-0)^2 + (y-2)^2 = 2^2$

↓

circle of radius 2 centered at $(0,2)$.

(x,y)



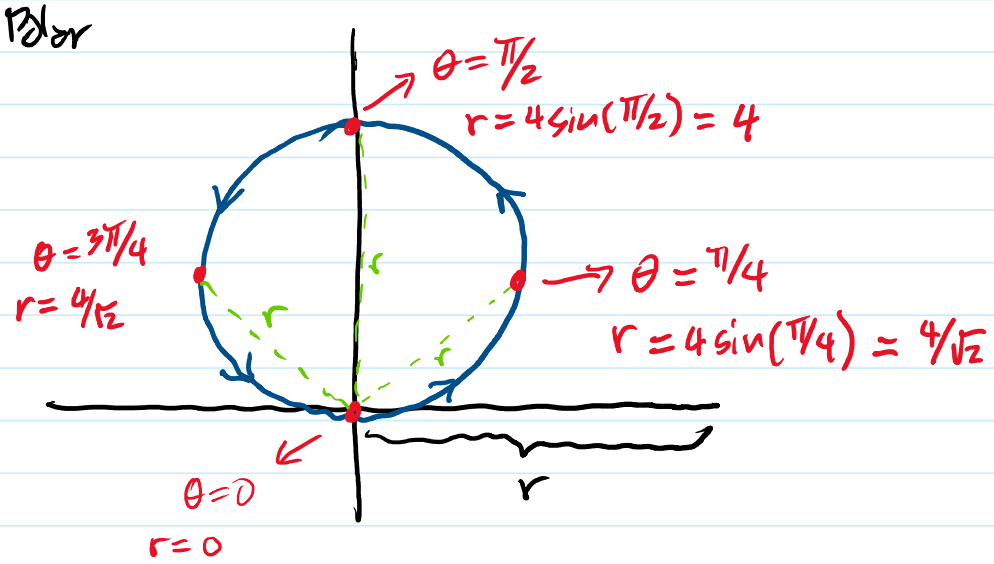
$a(y+d)^2 + e$

$d = \frac{b}{2a} = \frac{-4}{2} = -2$

$e = c - \frac{b^2}{4a} = -\frac{(-4)^2}{4} = -4$

$(y-2)^2 - 4$

Polar



- $r = 6\cos(\theta) - 8\sin(\theta)$

$r(r) = r(6\cos(\theta) - 8\sin(\theta))$

$r^2 = 6(r\cos(\theta)) - 8(r\sin(\theta))$

$x^2 + y^2 = 6x - 8y$

$(x^2 - 6x) + (y^2 + 8y) = 0$

↓

$(x^2 - 6x + 9 - 9) + (y^2 + 8y + 16 - 16) = 0$ ← "adding zero" to complete the square

$(x^2 - 6x + 9) + (y^2 + 8y + 16) = 9 + 16$

$(x^2-6x+9-9)+(y^2+8y+16-16)=0$ ← adding zero to complete the square

$(x^2-6x+9)+(y^2+8y+16)=9+16$

↓

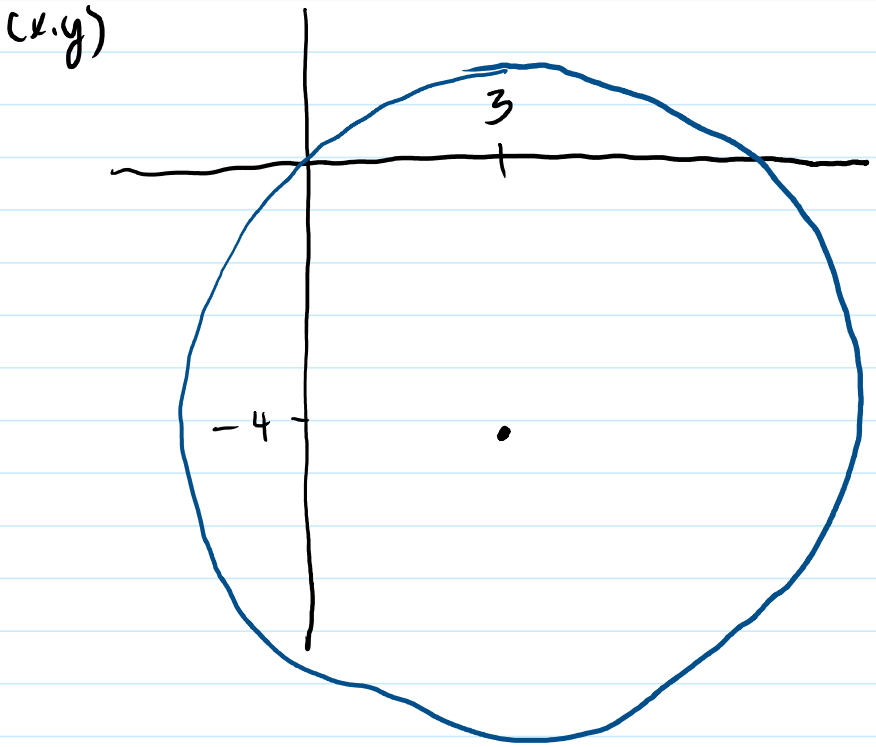
$(x-3)^2+(y+4)^2=25$

↓

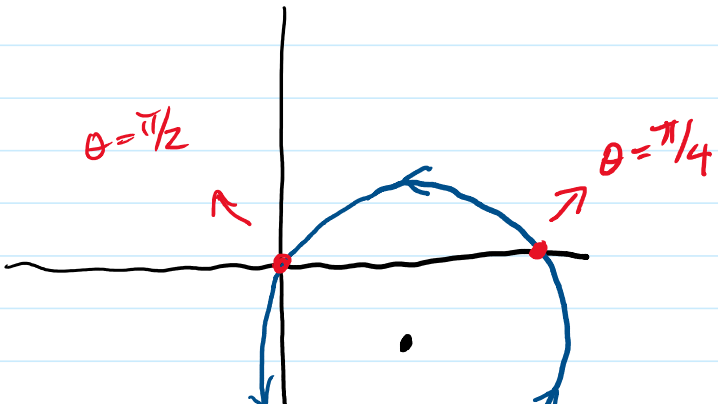
$(x-3)^2+(y+4)^2=5^2$

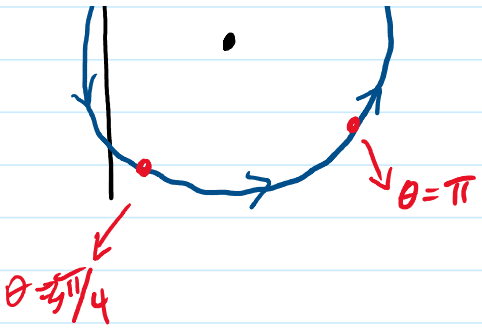
↓

this is a circle of radius 5 centered at $(3,-4)$



polar





Mini-Activity

1. Rewrite the equation $r = \sec\theta\tan(\theta)$ into (x,y) coordinates and graph it using desmos for both (x,y) & polar.

2. Try out polar graphs:

   a. $\theta = K$ → line

   b. $r = a\cos(\theta) + b\sin(\theta)$ → circle

   c. $r = a + b\theta$ → spiral

   d. $r = a(1+\cos\theta)$
      $r = a(1-\cos\theta)$
      $r = a(1+\sin(\theta))$
      $r = a(1-\sin\theta)$
      } cardoids

   e. $r = 2\cos(\theta) + b$
      $r = 2\sin(\theta) + b$
      } Limaçon

   f. $r = 2\cos(b\theta)$
      $r = 2\sin(b\theta)$
      } rose

Use the equations above to draw something Holiday using Desmos.