Basis

Objectives:

1. Introducing change of basis
2. Introducing linear transformations (on the same vector space).

Recall: <u>Basis Definition</u>

A basis set of vectors must 1. linearly independent
2. span the vector space

Changing Basis Vectors

Example: Let $B = \{\vec{b_1}, \vec{b_2}\}$ be as set of basis vectors.

$$b_1 = \begin{bmatrix} \sqrt{2}/2 \\ -\sqrt{2}/2 \end{bmatrix} \text{ and } b_2 = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix} \xrightarrow{\text{matrix}} A = \begin{array}{c} \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \\ \begin{array}{cc} \vec{b_1} & \quad \vec{b_2} \end{array} \end{array}$$

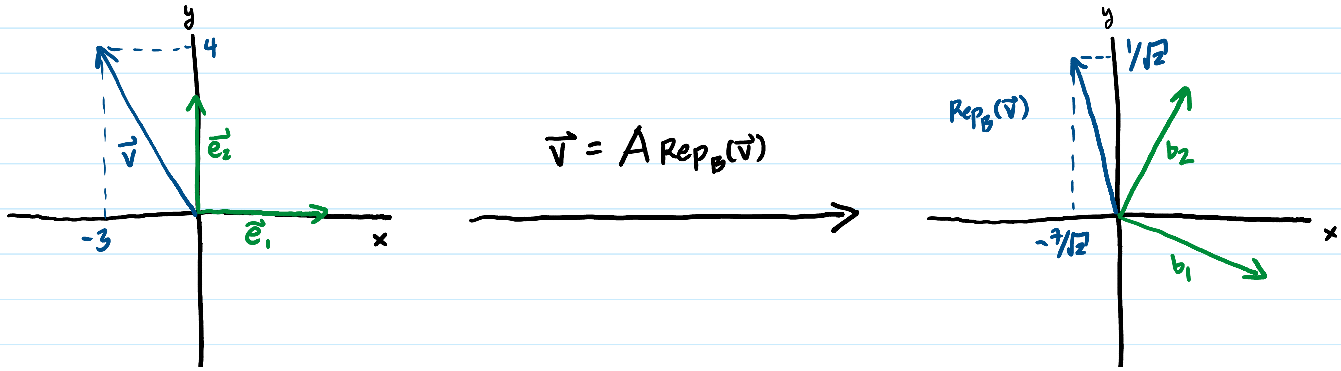
these are basis vectors because its linearly independent and span the vector space.
Suppose we have a vector using the standard basis $\vec{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$.

We want to find a vector that changes the coordinates of $\vec{v}$ onto the basis B.

$$\overset{\vec{v}}{\begin{bmatrix} -3 \\ 4 \end{bmatrix}} = \overbrace{\begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}}^{A} \overset{Rep_B(\vec{v})}{\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}} \left.\vphantom{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}\right\} \begin{array}{l} \text{two equations} \\ \text{two unknowns} \end{array}$$

$$\begin{aligned} -3 &= \sqrt{2}/2\, c_1 + \sqrt{2}/2\, c_2 \\ 4 &= -\sqrt{2}/2\, c_1 + \sqrt{2}/2\, c_2 \end{aligned} \longrightarrow \begin{aligned} c_1 &= -7/\sqrt{2} \\ c_2 &= 1/\sqrt{2} \end{aligned}$$

the new vector is $\begin{bmatrix} -7/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}_B = Rep_B(\vec{v})$.



Linear Transformations

A **transformation** $T: V \longrightarrow W$ that maps vectors from a vector space V to a diffirent (or the same) vector space W while preserving addition and scalar multiplication.

A **change-of-basis** transformation is a special case.