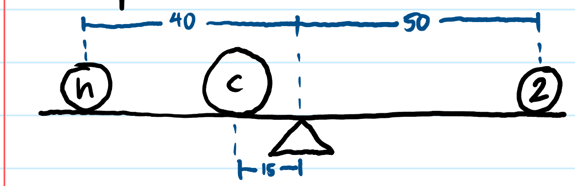
Gauss's Method

Objectives:

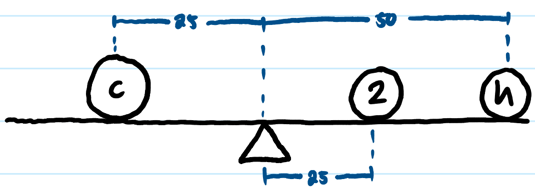
1. Introduce the Gaussian Elimination method.
2. Use Gauss's method to solve systems of equations.

*** Example: Statics**



System of Equations

$$40h + 15c = 50(2)$$
$$25c = 25(2) + 50h$$



Solving for h and c:

$$\begin{aligned} 40h + 15c &= 100 \quad R_1 \\ -50h + 25c &= 50 \quad R_2 \end{aligned}$$

$\xrightarrow{R_1 = (1/40)R_1}$

$$\begin{aligned} h + (15/40)c &= 100/40 \\ -50h + 25c &= 50 \end{aligned}$$

$\xrightarrow{R_2 = (50)R_1 + R_2}$

$$\begin{aligned} h + (15/40)c &= 100/40 \\ 0h + 50(15/40)c + 25c &= 50(100/40) + 50 \end{aligned}$$

$$\begin{aligned} h + 3/8\, c &= 5/2 \\ 175/4\, c &= 175 \end{aligned}$$

$\xrightarrow{R_2 = (4/175)R_2}$

$$\begin{aligned} h + 3/8\, c &= 5/2 \\ c &= 4 \end{aligned}$$

$\downarrow$ back substitution

$$\begin{aligned} h + 3/8(4) &= 5/2 \\ h &= 1 \end{aligned}$$

Thus, the solution is $h = 1$ & $c = 4$.

Theorem (Gauss's Method): If a linear system is changed to another by one of these operations

(1) an equation is swapped with another

(2) an equation has both sides multiplied by a nonzero constant

(3) an equation is replaced by the sum of itself and a multiple of another

then the two systems have the same set of solutions.

* Example: 3 eq., 3 unk.

$$\begin{array}{rl} x + y \qquad = 0 & R_1 \\ 2x - y + 3z = 3 & R_2 \\ x - 2y - z = 3 & R_3 \end{array} \xrightarrow{\text{swap } R_2 \text{ \& } R_3} \begin{array}{l} x + y \qquad = 0 \\ x - 2y - z = 3 \\ 2x - y + 3z = 3 \end{array}$$

$$\xrightarrow{R_2 = (-1)R_1 + R_2} \begin{array}{l} x + y \qquad = 0 \\ 0 - 3y - z = 3 \\ 2x - y + 3z = 3 \end{array}$$

$$\xrightarrow{R_2 = (-1/3) R_2} \begin{array}{l} x + y \qquad = 0 \\ \quad y - (1/3) z = -1 \\ 2x - y + 3z = 3 \end{array}$$

$$\xrightarrow{R_3 = (-2)R_1 + R_3} \begin{array}{l} x + y \qquad = 0 \\ \quad y - (1/3) z = -1 \\ 0 - 3y + 3z = 3 \end{array}$$

$$\xrightarrow{R_3 = 3R_2 + R_3} \begin{array}{l} x + y \qquad = 0 \\ \quad y - (1/3) z = -1 \\ \quad 0 - z = 0 \end{array}$$

$\downarrow$ back substitution

$$\left.\begin{array}{l} z = 0 \\ y = -1 \\ x = 1 \end{array}\right\} \text{solution}$$