Row Reduction

Objectives:

1. Introduce the Gauss-Jordan elimination method.
2. Solve linear system using Gauss-Jordan's method.

* Example : Solving for h and c:

$$\left.\begin{aligned} 40h + 15c &= 100 \\ -50h + 25c &= 50 \end{aligned}\right\}$$ system of equations from the statics example

augmented matrix

$$\left[\begin{array}{cc|c} 40 & 15 & 100 \\ -50 & 25 & 50 \end{array}\right]$$

Gaussian Elimination

$$\begin{array}{c c} & \begin{array}{ccc} h & c & \end{array} \\ \begin{array}{c} R_1 \\ R_2 \end{array} & \left[\begin{array}{cc|c} 1 & 3/8 & 5/2 \\ 0 & 1 & 4 \end{array}\right] \end{array}$$

row echelon form

Gauss-Jordan Elimination

$R_2 = -3/8 R_2 + R_1$

$$\left[\begin{array}{cc|c} 1 & & 5/2 \\ 0 & 1 & 4 \end{array}\right]$$

$$\begin{array}{c} \begin{array}{ccc} h & c & \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 4 \end{array}\right] \end{array}$$

reduced row echelon form (identity matrix)

$h = 1$
$c = 4$

* Example:

$$\begin{aligned} x + y \quad\quad &= 0 \\ 2x - y + 3z &= 3 \\ x - 2y - z &= 3 \end{aligned}$$

augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 2 & -1 & 3 & 3 \\ 1 & -2 & -1 & 3 \end{array}\right]$$

Gaussian Elimination

$$\begin{array}{c c} & \begin{array}{cccc} x & y & z & \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & -1/3 & -1 \\ 0 & 0 & -1 & 0 \end{array}\right] \end{array}$$

row echelon form

Gauss-Jordan Elimination

Gauss-Jordan Elimination

$$\xrightarrow{R_3 = -R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & -1/3 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 = 1/3 R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 = -R_2 + R_1} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

reduced row echelon form (identity)

$x = 1$
$y = -1$
$z = 0$

Example: $2x + y - z = 3$
$x - 4y + 2z = 1$

$$\xrightarrow{\text{augmented matrix}} \begin{array}{c} \left[\begin{array}{ccc|c} 2 & 1 & -1 & 3 \\ 1 & -4 & 2 & 1 \end{array}\right] \end{array} \begin{array}{c} R_1 \\ R_2 \end{array}$$

$$\xrightarrow{\text{swap } R_1 \text{ \& } R_2} \left[\begin{array}{ccc|c} 1 & -4 & 2 & 1 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$$\xrightarrow{R_2 = -2R_1 + R_2} \left[\begin{array}{ccc|c} 1 & -4 & 2 & 1 \\ 0 & 9 & -5 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 = (-1/9) R_2} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & -4 & 2 & 1 \\ 0 & 1 & -5/9 & 1/9 \end{array}\right] \end{array}$$

row echelon form

$$\xrightarrow{R_1 = 4R_2 + R_1} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -2/9 & 13/9 \\ 0 & 1 & -5/9 & 1/9 \end{array}\right] \end{array}$$

reduced row echelon form

Let $z = s$. So, $x - 2/9\, s = 13/9$
$y - 5/9\, s = 1/9$

$x = 13/9 + 2/9\, s$
$y = 1/9 + 5/9\, s$
$z = s$

$y = 1/9 + 5/9\, s$

$z = s$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13/9 \\ 1/9 \\ 0 \end{bmatrix} + \begin{bmatrix} 2/9 \\ 5/9 \\ 1 \end{bmatrix} s$$