Continuous Random Variables (CRVs)

Tuesday, October 25, 2022

Recall: Two types of Random Variables

	Discrete countable finite & infinite	Continuous uncountable finite & infinite
sample space		
probability functions ↓	prob. mass functions	prob. density functions
Definitions	1. $p(x) \geq 0$ for all x 2. $\sum_x p(x) = 1$	1. $f(x) \geq 0$ for all x 2. $\int_{\mathbb{R}} f(x)\,dx = 1$
cumulative distribution functions	$F_X(x) = \sum_{u=-\infty}^{x} p(u)$	$F_X(x) = \int_{-\infty}^{x} f(u)\,du$
Expected value	$E[X] = \sum_x x p(x)$	$E[X] = \int_{\mathbb{R}} x f(x)\,dx$
moment generating functions	$M_X(s) = E[e^{sX}] = \sum_{u=0}^{\infty} E[X^k]\frac{s^k}{k!}$	$\longrightarrow M_X(s) = E[e^{sX}] = \int_{\mathbb{R}} e^{tx} f(x)\,dx$
moments	$E[X^k] = \frac{d^k}{ds^k} M_X(s)\Big\|_{s=0}$	
	$E[X^k] = \sum_{u=0}^{\infty} x^k p(x)$	$E[X^k] = \int_{\mathbb{R}} x^n f(x)\,dx$
Variance	$Var(X) = E[X^2] - (E[X])^2$	
stand. dev.	$\sigma(X) = \sqrt{Var(X)}$	